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CONVECTIVE ADJUSTMENT IN BAROCLINIC ATMOSPHERES

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Local convection in planetary atmospheres is generally considered to result from the action of gravity on small regions of anomalous density. We show that in rotating baroclinic fluids the total potential energy for small-scale convection contains a centrifugal as well as a gravitational contribution. Convective adjustment in such an atmosphere results in the establishment of near adiabatic lapse rates of temperature along suitably defined surfaces of constant angular momentum, rather than in the vertical. This leads in general to sub-adiabatic vertical lapse rates. We demonstrate by example that such an adjustment actually occurs in the earth's atmosphere and estimate the magnitude of the effect for several other planetary atmospheres.

Among the more important processes operative in many planetary and stellar atmospheres is small-scale bouyant convection, whose presence is often inferred from the observation of adiabatic lapse rates of temperature, a condition which both theory and experiment show will result from the action of convection at very high Rayleigh number. Conversely, the observation of sub-adiabatic lapse rates appears to rule out the action of bouyant convection. The purpose of this presentation is to demonstrate that small-scale convection in rotating baroclinic flows is driven by centrifugal as well as gravitational bouyancy and results in the establishment of adiabatic lapse rates along locally defined angular momentum surfaces and sub-adiabatic lapse rates in the vertical. We estimate the magnitude of this effect in planetary atmospheres and discuss a technique for inferring the lapse rates along angular momentum surfaces from vertical profiles of wind and temperature. This work represents a generalization to planetary atmospheres of the theory of slantwise convection in the earth's atmosphere developed by Emanuel (1983a,b), and may be considered a nonlinear extension of the theory of symmetric instability developed by Solberg (1933), Stone (1966), and others.

Consider a geostrophically and hydrostatically balanced baroclinic flow on a segment of a sphere sufficiently small that the variation of the local normal component of the planetary rotation may be neglected. Figure 1 shows a hypothetical distribution of isotherms on a constant pressure surface in such a segment. In order that the flow remain close to geostrophic, the length and velocity scales associated with the pattern must be such that the Rossby number, defined $R_0 \equiv U_0/fL$ is much less than unity. Here U_0 and L are velocity and length scales, and f is the Coriolis parameter. It is helpful to define a local coordinate system which has its x-axis parallel to the isotherms, as illustrated in Fig. 1.

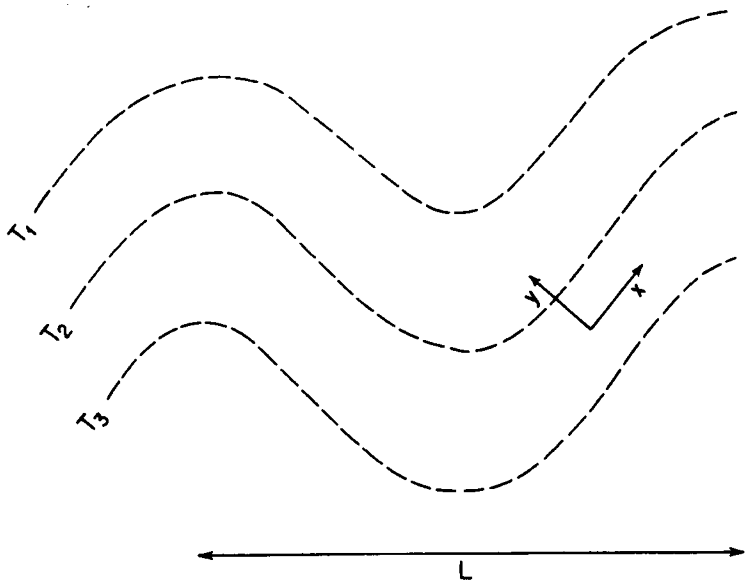

Figure 1. Definition of coordinate system with respect to large-scale pattern of isotherms on an isobaric surface.

In this coordinate system, the inviscid equations of motion may be written

$$\frac{du}{dt} = fv \equiv f\frac{dy}{dt} , \tag{1}$$

$$\frac{dv}{dt} = f(u_g - u) \tag{2}$$

where u and v are the components of velocity in the x and y directions and u_g is the geostrophic velocity. Equation (1) applies as long as the parcel remains in a flow for which pressure gradients parallel to x are small. By virtue of (1), a quantity M, defined

$$M \equiv u - fy \tag{3}$$

is conserved following the motion of fluid parcels and (1) and (2) may be rewritten

$$\frac{dM}{dt} = 0 , \tag{4}$$

$$\frac{dv}{dt} = -f(M-M_g), \tag{5}$$

where M_g is the geostrophic value of M. The quantity M represents a linearization of the angular momentum per unit mass of the fluid.

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It has been demonstrated by Emanuel (1983a) that provided the ensuing convection is characterized by length scales along the total acceleration vector that are large compared to length scales normal to this vector, the effect of the convection on the pressure field may be neglected so that M_g may be regarded as a fixed function of space in (5). Similarly, if the temperature perturbations are small compared to the mean temperature, the inviscid equation of vertical motion may be approximated by

$$\frac{dw}{dt} = \frac{g}{\theta}(\theta - \theta_a), \tag{6}$$

where g is the acceleration of gravity, θ is the potential temperature and θ_a is the ambient potential temperature, also a fixed function of space. The thermal wind equation relates M_g to θ_a:

$$f\frac{dM_g}{dz} = -\frac{g}{\theta_a}\frac{\partial\theta_a}{\partial y}, \tag{7}$$

The system is closed by the equation for the parcel potential temperature:

$$c_p \frac{d \ln \theta}{dt} = \frac{\dot{H}}{T} \tag{8}$$

where c_p is the heat capacity at constant pressure, $\dot{H}$ is the diabatic heating rate, and T is the absolute temperature. The diabatic heating includes effects of radiative transfer and latent heat release associated with phase changes. The latter will generally result in a parcel potential temperature which is a function of pressure. We note that (6) does not include effects due to changing proportions of chemical constituents or to the presence of suspended liquids or solids.

Note that equations (5) and (6) for the lateral and vertical accelerations are formally identical in form as they both depend on the differences between conserved parcel quantities and fixed ambient properties. This is simply to say that there is no qualitative difference between centripetal and gravitational acceleration. In a baroclinic flow, M_g varies with height so that, according to (5), vertically displaced parcels will always suffer lateral as well as vertical accelerations; convection in this case will therefore be "slantwise" rather than vertical. An extensive discussion of the nature of slantwise convection in the earth's atmosphere may be found in Emanuel (1983a,b); here we summarize and extend the main results as they pertain to convection in planetary atmospheres.

The relations (5) and (6) imply that unstable convection will occur if a displacement results in accelerations in the same directions as the displacement; they also imply that the parcel will come to rest at or oscillate about stable

equilibrium points where $M = M_g$ and $\theta = \theta_a$. The total potential energy available to drive parcels between their initial unstable equilibrium points and their final stable equilibria is the path integral of the vector force (per unit mass) $\underset{\sim}{F}$:

$$\mathrm{TPE} = \int_1^2 \underset{\sim}{F} \cdot d\ell = \int_1^2 \left\{-f(M-M_g)\hat{j} + \frac{g}{\theta}(\theta-\theta_a)\hat{k}\right\} \cdot d\ell \ , \tag{9}$$

where TPE is the "total potential energy" and (5) and (6) have been substituted as the lateral and vertical components of the force per unit mass. The unit vectors in y and z are $\hat{j}$ and $\hat{k}$.

Now by virtue of (7), the force vector is irrotational:

$$\nabla \times \underset{\sim}{F} = 0 \ ,$$

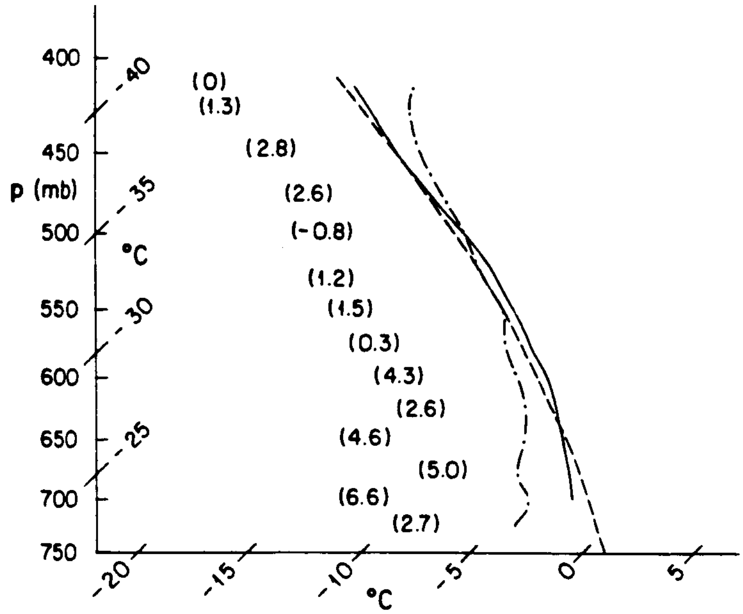
so that the choice of integration path is immaterial. For convenience, we choose a path along a surface of constant M and redefine the potential energy:

$$\mathrm{TPE} = \int_1^2 \Big|_{M_g} \frac{g}{\theta}(\theta-\theta_a)dz \ , \tag{10}$$

which simply means that the total potential energy for small-scale convection is the integral of the parcel bouyancy as it is lifted along surfaces of constant geostrophic pseudo-angular momentum, M_g. (We note that this in no way implies that the ensuing convection will result in motions along M_g surfaces.) This definition of potential energy represents a simple generalization of the classical expression which is identical to (10) except that the path of integration is in the vertical direction. In a barotropic flow, M_g surfaces are vertical and (10) reduces to the classical expression. The important implication of (10) for our present purpose is that the state of neutrality to small-scale convection is characterized by adiabatic lapse rates along M_g surfaces. That this type of adjustment actually occurs in the earth's atmosphere is illustrated in Fig. 2, which shows a temperature sounding measured by an instrumented aircraft flying down an M surface, a vertical sounding, and a pseudo-adiabat for processes involving phase change of water between vapor and ice. While the vertical sounding is definitely sub-adiabatic, the M-surface sounding is almost perfectly adiabatic. An interpretation of the convective dynamics of this atmosphere based only on the vertical sounding would be most misleading. If the vertical sounding measures horizontal wind as well, however, it is possible to roughly estimate the M_g-surface temperature structure from the vertical sounding.

Suppose that a baroclinic flow is characterized by zonal winds which increase with height. Then, as illustrated in Fig. 3, M_g surfaces will slope poleward with increasing height. We estimate the temperature structure along a particular M_g surface using the thermal wind equation and geometry.

Figure 2. Earth atmospheric temperature soundings shortly after midnight Eastern Standard Time on 16 November 1983. Solid line: Aircraft sounding along M surface from central Maine to coastal New Hampshire. Departures of M ($m\ s^{-1}$) from value at 412 mb indicated in parentheses at left. Dash-dot line: Vertical sounding made by spiral ascent of aircraft at Brunswick, Maine. Dashed line: Moist adiabat for vapor-ice transition corresponding to ice equivalent potential temperature of 315 K.



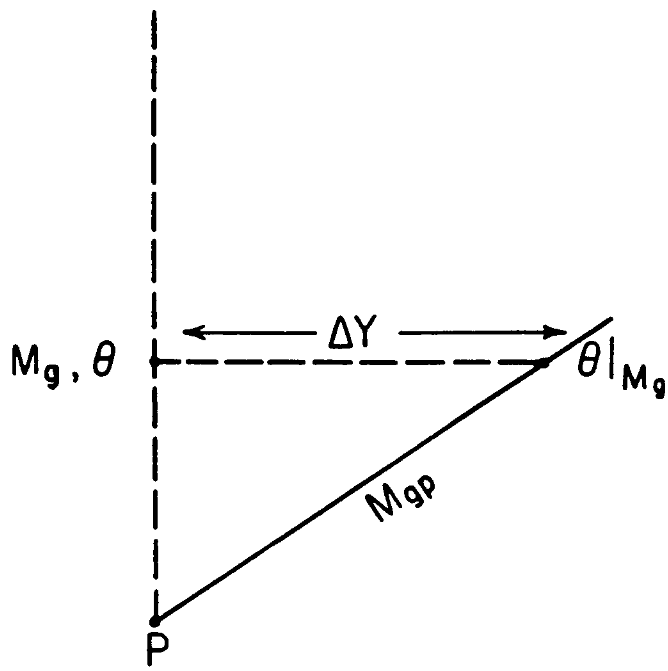


Figure 3. Geometry used to estimate distribution of potential temperature along the M_g surface. See text for explanation.

It is first necessary to define a level p at which we wish to examine the stability with respect to overlying fluid. This is important when phase changes occur, since the shape of an adiabat will depend on the temperature, pressure, and chemical composition of the starting parcel. When there are no phase changes, the choice of this level is arbitrary. We now wish to estimate the temperature structure along an M_g surface intersecting p from a vertical sounding which also passes through p. At an arbitrary height the potential temperature along the M_g surface is related to that of the vertical sounding by

$$\ln\theta\Big|_{M_g} = \ln\theta + \Delta Y \frac{\partial \ln\theta}{\partial y} \tag{11}$$

where ΔY is the horizontal distance between the M_g surface and the vertical sounding. Provided the horizontal gradient of M_g is constant this distance is given by

$$\Delta Y = \frac{\Delta M_g}{\partial M_g/\partial y} \tag{12}$$

where ΔM_g is the difference between the M_g at the vertical sounding and the M_g characterizing the parcel level p. From the definition of M, (3),

$$\Delta M_g = U_{gp} - U_g$$

and (13)

$$\frac{\partial M_g}{\partial y} = -\eta_g$$

where η_g is the vertical component of absolute geostrophic vorticity, and U_{gp} is the geostrophic wind at level p. Substituting the relations (13) into (12) and (12) into (11), and using (7), there results

$$\ln\theta\Big|_{M} = \ln\theta - \frac{1}{g}\frac{f}{\eta_g}\frac{\partial U_g}{\partial z}(U_g - U_{gp}). \tag{14}$$

Given an estimate of η_g, this may be used to directly infer the temperature structure along a particular M_g surface from a vertical sounding of θ and U, provided one is willing to approximate U_g by U. If phase changes are not important, stability can be assessed by infinitesimal displacements from level p and (14) can be used as a linear stability estimate:

$$\left.\frac{\partial \ln\theta}{\partial z}\right|_{M_g} = \frac{\partial \ln\theta}{\partial z} - \frac{1}{g}\frac{f}{\eta_g}\,(\partial U_g/\partial z)^2 \tag{15}$$

In this case, generalized convective neutrality occurs when

$$Ri \equiv \frac{g\partial \ln\theta/\partial z}{(\partial U_g/\partial z)^2} = f/\eta_g \ ,$$

where Ri is the Richardson number. This is nearly identical to the linear neutral condition derived by Stone (1966).

When phase changes drive the convection, parcels may be stable to small displacements but unstable for finite displacements. Here it is important to use (14) to assess the stability of the flow with respect to particular parcels. We note that if (14) is substituted into (10) for the total potential energy there results

$$TPE = \frac{1}{2}\frac{f}{\eta_g}\,(U_{g_2} - U_{g_1})^2 + \int_1^2 \Big|_y \frac{g}{\theta}\,(\theta - \theta_a)dz \ , \tag{16}$$

if η_g is constant. The total potential energy is the sum of the bouyant energy of a vertically lifted parcel and the available kinetic energy of the geostrophic wind.

Now we will estimate vertical lapse rates in convectively adjusted planetary atmospheres. We have argued that generalized convective adjustment will result in adiabatic lapse rates along surfaces of constant M_g. If phase changes are unimportant, this state will be characterized by zero lapse rates of potential temperature along M_g surfaces.* According to (15), then, the vertical lapse rate will be given by

$$\frac{\partial \ln\theta}{\partial z} = \frac{1}{g}\frac{f}{\eta_g}\,(\partial U_g/\partial z)^2 \tag{17}$$

*If phase changes are not negligible, then the arguments which follow will still be valid if θ is replaced by $\theta_e{}^*$, where the latter is the temperature a parcel would have if a) it were saturated with the constituent whose phase change is being considered and b) it were then lifted first to zero pressure, all condensate removed, and then brought adiabatically to a reference pressure.

Using (7), this may alternatively be expressed

$$\frac{\partial \ln\theta}{\partial z} = \frac{g}{f\eta_g}\left(\frac{\partial \ln\theta}{\partial y}\right)^2 \qquad (18)$$

A nondimensional measure of the significance of this lapse rate is formed by comparing it to the dry adiabatic lapse rate of absolute temperaure (g/c_p):

$$\chi \equiv \frac{\partial\theta/\partial z}{g/C_p} \simeq \frac{c_p\theta}{g^2}(\partial U_g/\partial z)^2 \quad \text{or} \quad \frac{c_p}{\theta f^2}\left(\frac{\partial\theta}{\partial y}\right) \qquad (19)$$

where we have used f as an estimate of η_g. Table 1 shows estimates of χ for the five major planets for which reasonable data were available to the author. The estimates are based on mean conditions in the tropospheres of the planets. Of the five planets, only Earth and Mars may be expected to show significant departures of vertical lapse rates from adiabatic values within convectively adjusted layers.

Table 1

Estimates of χ for the Tropospheres of Five Planets

	χ
Venus*	2×10^{-2}
Earth	3×10^{-1}
Mars	4×10^{-1}
Jupiter	5×10^{-4}
Saturn	5×10^{-4}

*Because the large-scale flow on Venus is in cyclostrophic rather than geostrophic balance, the above relation for χ in terms of the horizontal potential temperature gradient will be modified for this application. A straightforward analysis of the cyclostrophic balance equation suggests, however, that the expression for χ in terms of the vertical wind shear is the same, to order of magnitude, as the result derived above.

To summarize, convective adjustment in rotating baroclinic fluids is brought about by the action of both gravitational and centrifugal accelerations. The net effect of the adjustment is to drive temperature lapse rates toward adiabatic values along surfaces of constant geostrophic angular momentum; these surfaces are vertical in a barotropic flow but have a finite slope under baroclinic conditions. We have presented observations which demonstrate that moist slantwise adjustment does indeed occur in the earth's atmosphere, and developed a method for inferring M_g surface temperature structure from vertical soundings. Estimates of the departure of vertical lapse rates from adiabatic values within convectively adjusted regions of five planetary atmospheres show that the effect is significant only in the tropospheres of Earth and Mars.

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DR. STONE: Are there comments? Bill Rossow?

DR. ROSSOW: I have two comments and I think I'll do them in reverse-sensible order to get the first comment in. The point you made to start with, that the radiative time scale is very long compared to the convective adjustment time also raises another whole class of phenomena, namely that if you were to try to run a numerical model with that condition in it, which I've had the pleasure of doing, what you find is that it almost never convects. It only does so every third Thursday under a blue moon because that's all it needs to do to satisfy the balance. What happens in that case is that the large scale dynamics, which are busy doing other things like taking care of horizontal temperature gradients, control the static stability, and the convection doesn't really matter much. That may be a possible problem for all these kinds of puzzles, that we're not really looking at something which is smoothly balanced, that the processes are going on all the time. The question I had was, does your analysis essentially say that small scale convection of the usual kind should maintain the entropy state that Andy starts out assuming as his basic state?

DR. EMANUEL: I'll have to take your comment before your question. I suppose in a sense it depends on how you define convective time scales. For example, if we really did have a radiative convective equilibrium, we would compare something like the radiative time scale with a time scale characterizing the

fluxes and heat in convection. What I really wish to point out is that some, say, viscously determined time scale of convection has to be very very quick. So in these circumstances you would imagine actually seeing a lapse rate close to adiabatic. So far as the initial condition Andy Ingersoll used, I'm not sure I exactly understand the initial condition, but if it's a spherical rotation, if there are no perturbations on that, if you're starting off with just solid body rotation, then of course angular momentum surfaces are cylinders, and then if you postulate that you have convective equilibrium then it's quite proper to take the entropy to be constant along cylinders. They won't stay that way if the cylinders are deformed by the motion that develops.

DR. ROSSOW: It sounded to me that what you were really saying, your sort of definition of total potential, is that the proper direction for neutral bouyant stability in a rotating fluid is along something like the rotation axis and that sounded like where Andy Ingersoll started.

DR. EMANUEL: Yes, that's right. My only point is that that's true, but it's along the total vorticity vector including the vorticity of the large scale flow if it exists, not just the planetary vorticity.

DR. FELS: I'm sorry, I'm very confused about the relationship between this and the conventional old fashioned vertical stabilization by baroclinic instability. Now is this a reworking or is this on a much smaller scale?

DR. EMANUEL: No, it isn't. In fact, I should make a very strong distinction between what I'm talking about and what is conventionally called baroclinic instability. There's a tremendous amount of confusion on this problem. I'm really talking about a kind of convection which has been called symmetric instability. I guess I have to disagree with Andy Ingersoll on his characterization of baroclinic instability as sloping convection, although he's not the first to have done that; I think that Hide in fact characterized it that way as well, because in fact baroclinic instability is trying to redistribute potential vorticity principally. In my case we're dealing with something that's fundementally trying to redistribute heat but I'm only claiming it's redistributing heat along angular momentum surfaces.

DR. FELS: You're talking about inertial instability here.

DR. EMANUEL: Baroclinic inertial instability, yeah.

DR. STONE: Larry Trafton.

DR. TRAFTON: How would you characterize the latitudinal dependence of the degree of subadiabaticity? Would it disappear at the equator for example?

DR. EMANUEL: Yes, it would have to disappear at the equator and presumably it would have to disappear, well, it's not true that it has to disappear at the pole for large scale asymmetries which, for example, cause flow across the pole. That happens in the Earth's atmosphere. But I would suppose that on the average it would be maximum somewhere in the middle latitudes, so you would expect on the average the most sub-adiabatic lapse rates somewhere in the middle latitudes.

DR. ALLISON: One more thing about your results. Can you tell me how to re-express your statification parameter χ in terms of the Richardson number?

DR. EMANUEL: Uh, let me think about that. Essentially, the adjustment problem can be phrased as making the Richardson number close to unity everywhere. But that doesn't tell you the answer to the question of how sub-adiabatic the fluid is, because that depends on the individual magnitudes of the horizontal temperature gradient and the stratification. So you can't really express the sub-adiabaticity just in terms of the Richardson number. You need something else as well.

DR. STONE: I have a plea for one more question from Mike Belton.

DR. BELTON: It's probably silly, but there is one way of maybe getting to lapse rates deeper down and that is to look for silly results for abundances using thermal spectroscopy. So for example in Uranus we have a problem with ammonia and one of the ideas that was being kicked around was that maybe there was a sub-adiabatic region deep down in the atmosphere which seemed ridiculous. Then yesterday we heard "Well maybe there's no water on Jupiter, or very little water," but that could be interpreted possibly as a sub-adiabatic lapse rate in that region, say at 3 bars, 2 bars. The problem with it, and probably why it's silly, is that it requires an enormous sub-adiabaticity, and that doesn't seem to be in the cards.

DR. EMANUEL: Not unless there are extremely large horizontal temperature gradients...

DR. STONE: Alright. Now I think we can adjourn. Thank you all.

During the lunch break on the second day of the meeting several scientists attended a tour of the nearby Cathedral of St. John the Divine led by conference participant Canon Jonathan King. The tour included a review of a stained glass window depicting the planets, excuted by Ernest W. Lakeman in 1934. The slanted equatorial feature rendered for Jupiter was interpreted by Dr. Conway Leovy as an example of an atmospheric breaking wave or "surf zone" as a part of his review presentation for the afternoon session. (The illustration here has been reproduced from an original color photograph by Gregory Thorp by permission of The Cathedral Shop; 1047 Amsterdam Avenue; New York, NY.)